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3-1 Study Guide and Intervention

Solving Systems of Equations by Graphing

Graph Systems of Equations A system of equations is a set of two or more equations containing the same variables. You can solve a system of linear equations by graphing the equations on the same coordinate plane. If the lines intersect, the solution is that intersection point.

Example **Solve the system of equations by graphing.** $\mathbf{x - 2y = 4}$, $\mathbf{x + y = -2}$

Write each equation in slope-intercept form.

$x - 2y = 4 \quad \rightarrow \quad y = \frac{x}{2} - 2$

$x + y = -2 \quad \rightarrow \quad y = -x - 2$

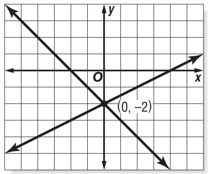
The graphs appear to intersect at (0, −2).

CHECK Substitute the coordinates into each equation.

$x - 2y = 4$; $0 - 2(-2) \stackrel{?}{=} 4$; $4 = 4$ ✓

$x + y = -2$; $0 + (-2) \stackrel{?}{=} -2$; $-2 = -2$ ✓

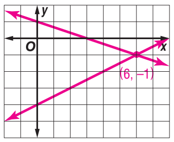
The solution of the system is (0, −2).



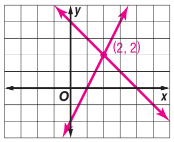
Exercises

Solve each system of equations by graphing.

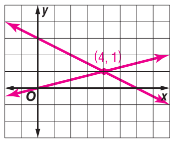
1. $y = -\frac{x}{3} + 1$; $y = \frac{x}{2} - 4$ **(6, −1)**



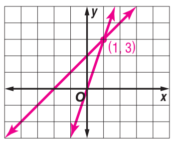
2. $y = 2x - 2$; $y = -x + 4$ **(2, 2)**



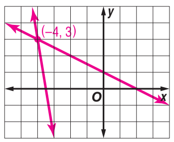
3. $y = -\frac{x}{2} + 3$; $y = \frac{x}{4}$ **(4, 1)**



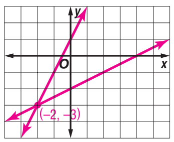
4. $3x - y = 0$; $x - y = -2$ **(1, 3)**



5. $2x + \frac{y}{3} = -7$; $\frac{x}{2} + y = 1$ **(−4, 3)**



6. $\frac{x}{2} - y = 2$; $2x - y = -1$ **(−2, −3)**



Lesson 3-1

NAME ______________________ DATE ________ PERIOD _____

3-1 Study Guide and Intervention *(continued)*

Solving Systems of Equations by Graphing

Classify Systems of Equations The following chart summarizes the possibilities for graphs of two linear equations in two variables.

Graphs of Equations	Slopes of Lines	Classification of System	Number of Solutions
Lines intersect	Different slopes	Consistent and independent	One
Lines coincide (same line)	Same slope, same *y*-intercept	Consistent and dependent	Infinitely many
Lines are parallel	Same slope, different *y*-intercepts	Inconsistent	None

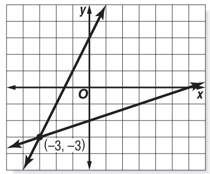
Example **Graph the system of equations and describe it as *consistent and independent*, *consistent and dependent*, or *inconsistent*.** $\mathbf{x - 3y = 6}$, $\mathbf{2x - y = -3}$

Write each equation in slope-intercept form.

$x - 3y = 6 \quad \rightarrow \quad y = \frac{1}{3}x - 2$

$2x - y = -3 \quad \rightarrow \quad y = 2x + 3$

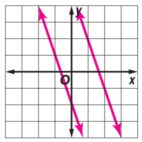
The graphs intersect at (−3, −3). Since there is one solution, the system is consistent and independent.



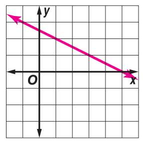
Exercises

Graph the system of equations and describe it as *consistent and independent*, *consistent and dependent*, or *inconsistent*.

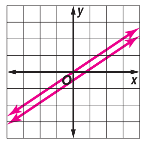
1. $3x + y = -2$; $6x + 2y = 10$ **inconsistent**



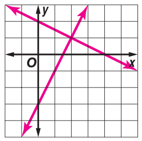
2. $x + 2y = 5$; $3x - 15 = -6y$ **consistent and dependent**



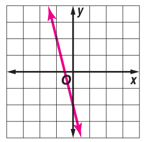
3. $2x - 3y = 0$; $4x - 6y = 3$ **inconsistent**



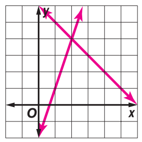
4. $2x - y = 3$; $x + 2y = 4$ **consistent and independent**



5. $4x + y = -2$; $2x + \frac{y}{2} = -1$ **consistent and dependent**



6. $3x - y = 2$; $x + y = 6$ **consistent and independent**



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3-2 Study Guide and Intervention
Solving Systems of Equations Algebraically

Substitution To solve a system of linear equations by **substitution**, first solve for one variable in terms of the other in one of the equations. Then substitute this expression into the other equation and simplify.

Example **Use substitution to solve the system of equations.** $2x - y = 9$; $x + 3y = -6$

Solve the first equation for y in terms of x.

$2x - y = 9$	First equation
$-y = -2x + 9$	Subtract 2x from both sides.
$y = 2x - 9$	Multiply both sides by −1.

Substitute the expression $2x - 9$ for y into the second equation and solve for x.

$x + 3y = -6$	Second equation
$x + 3(2x - 9) = -6$	Substitute 2x − 9 for y.
$x + 6x - 27 = -6$	Distributive Property
$7x - 27 = -6$	Simplify.
$7x = 21$	Add 27 to each side.
$x = 3$	Divide each side by 7.

Now, substitute the value 3 for x in either original equation and solve for y.

$2x - y = 9$	First equation
$2(3) - y = 9$	Replace x with 3.
$6 - y = 9$	Simplify.
$-y = 3$	Subtract 6 from each side.
$y = -3$	Multiply each side by −1.

The solution of the system is (3, −3).

Exercises

Solve each system of linear equations by using substitution.

1. $3x + y = 7$; $4x + 2y = 16$ — **(−1, 10)**
2. $2x + y = 5$; $3x - 3y = 3$ — **(2, 1)**
3. $2x + 3y = -3$; $x + 2y = 2$ — **(−12, 7)**
4. $2x - y = 7$; $6x - 3y = 14$ — **no solution**
5. $4x - 3y = 4$; $2x + y = -8$ — **(−2, −4)**
6. $5x + y = 6$; $3 - x = 0$ — **(3, −9)**
7. $x + 8y = -2$; $x - 3y = 20$ — **(14, −2)**
8. $2x - y = -4$; $4x + y = 1$ — **$\left(-\frac{1}{2}, 3\right)$**
9. $x - y = -2$; $2x - 3y = 2$ — **(−8, −6)**
10. $x - 4y = 4$; $2x + 12y = 13$ — **$\left(5, \frac{1}{4}\right)$**
11. $x + 3y = 2$; $4x + 12y = 8$ — **infinitely many**
12. $2x + 2y = 4$; $x - 2y = 0$ — **$\left(\frac{4}{3}, \frac{2}{3}\right)$**

Lesson 3-2

NAME ______________________ DATE __________ PERIOD _____

3-2 Study Guide and Intervention *(continued)*
Solving Systems of Equations Algebraically

Elimination To solve a system of linear equations by **elimination**, add or subtract the equations to eliminate one of the variables. You may first need to multiply one or both of the equations by a constant so that one of the variables has the same (or opposite) coefficient in one equation as it has in the other.

Example 1 **Use the elimination method to solve the system of equations.**

$2x - 4y = -26$
$3x - y = -24$

Multiply the second equation by 4. Then subtract the equations to eliminate the y variable.

$2x - 4y = -26$ → $2x - 4y = -26$
$3x - y = -24$ (Multiply by 4.) → $12x - 4y = -96$
$-10x = 70$
$x = -7$

Replace x with −7 and solve for y.

$2x - 4y = -26$
$2(-7) - 4y = -26$
$-14 - 4y = -26$
$-4y = -12$
$y = 3$

The solution is (−7, 3).

Example 2 **Use the elimination method to solve the system of equations.**

$3x - 2y = 4$
$5x + 3y = -25$

Multiply the first equation by 3 and the second equation by 2. Then add the equations to eliminate the y variable.

$3x - 2y = 4$ (Multiply by 3.) → $9x - 6y = 12$
$5x + 3y = -25$ (Multiply by 2.) → $10x + 6y = -50$
$19x = -38$
$x = -2$

Replace x with −2 and solve for y.

$3x - 2y = 4$
$3(-2) - 2y = 4$
$-6 - 2y = 4$
$-2y = 10$
$y = -5$

The solution is (−2, −5).

Exercises

Solve each system of equations by using elimination.

1. $2x - y = 7$; $3x + y = 8$ — **(3, −1)**
2. $x - 2y = 4$; $-x + 6y = 12$ — **(12, 4)**
3. $3x + 4y = -10$; $x - 4y = 2$ — **(−2, −1)**
4. $3x - y = 12$; $5x + 2y = 20$ — **(4, 0)**
5. $4x - y = 6$; $2x - \frac{y}{2} = 4$ — **no solution**
6. $5x + 2y = 12$; $-6x - 2y = -14$ — **(2, 1)**
7. $2x + y = 8$; $3x + \frac{3}{2}y = 12$ — **infinitely many**
8. $7x + 2y = -1$; $4x - 3y = -13$ — **(−1, 3)**
9. $3x + 8y = -6$; $x - y = 9$ — **(6, −3)**
10. $5x + 4y = 12$; $7x - 6y = 40$ — **(4, −2)**
11. $-4x + y = -12$; $4x + 2y = 6$ — **$\left(\frac{5}{2}, -2\right)$**
12. $5m + 2n = -8$; $4m + 3n = 2$ — **(−4, 6)**

Answers

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3-5 Study Guide and Intervention

Solving Systems of Equations in Three Variables

Systems in Three Variables Use the methods used for solving systems of linear equations in two variables to solve systems of equations in three variables. A system of three equations in three variables can have a unique solution, infinitely many solutions, or no solution. A solution is an **ordered triple**.

Example **Solve this system of equations.**

$$\begin{aligned} 3x + y - z &= -6 \\ 2x - y + 2z &= 8 \\ 4x + y - 3z &= -21 \end{aligned}$$

Step 1 Use elimination to make a system of two equations in two variables.

$$\begin{aligned} 3x + y - z &= -6 && \text{First equation} \\ (+)\ 2x - y + 2z &= 8 && \text{Second equation} \\ \hline 5x + z &= 2 && \text{Add to eliminate } y. \end{aligned}$$

$$\begin{aligned} 2x - y + 2z &= 8 && \text{Second equation} \\ (+)\ 4x + y - 3z &= -21 && \text{Third equation} \\ \hline 6x - z &= -13 && \text{Add to eliminate } y. \end{aligned}$$

Step 2 Solve the system of two equations.

$$\begin{aligned} 5x + z &= 2 \\ (+)\ 6x - z &= -13 \\ \hline 11x &= -11 && \text{Add to eliminate } z. \\ x &= -1 && \text{Divide both sides by 11.} \end{aligned}$$

Substitute −1 for x in one of the equations with two variables and solve for z.

$$\begin{aligned} 5x + z &= 2 && \text{Equation with two variables} \\ 5(-1) + z &= 2 && \text{Replace } x \text{ with } -1. \\ -5 + z &= 2 && \text{Multiply.} \\ z &= 7 && \text{Add 5 to both sides.} \end{aligned}$$

The result so far is $x = -1$ and $z = 7$.

Step 3 Substitute −1 for x and 7 for z in one of the original equations with three variables.

$$\begin{aligned} 3x + y - z &= -6 && \text{Original equation with three variables} \\ 3(-1) + y - 7 &= -6 && \text{Replace } x \text{ with } -1 \text{ and } z \text{ with 7.} \\ -3 + y - 7 &= -6 && \text{Multiply.} \\ y &= 4 && \text{Simplify.} \end{aligned}$$

The solution is (−1, 4, 7).

Exercises

Solve each system of equations.

1. $2x + 3y - z = 0$
$x - 2y - 4z = 14$
$3x + y - 8z = 17$
(4, −3, −1)

2. $2x - y + 4z = 11$
$x + 2y - 6z = -11$
$3x - 2y - 10z = 11$
$\left(2, -5, \frac{1}{2}\right)$

3. $x - 2y + z = 8$
$2x + y - z = 0$
$3x - 6y + 3z = 24$
infinitely many solutions

4. $3x - y - z = 5$
$3x + 2y - z = 11$
$6x - 3y + 2z = -12$
$\left(\frac{2}{3}, 2, -5\right)$

5. $2x - 4y - z = 10$
$4x - 8y - 2z = 16$
$3x + y + z = 12$
no solution

6. $x - 6y + 4z = 2$
$2x + 4y - 8z = 16$
$x - 2y = 5$
$\left(6, \frac{1}{2}, -\frac{1}{4}\right)$

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3-5 Study Guide and Intervention (continued)

Solving Systems of Equations in Three Variables

Real-World Problems

Example **The Laredo Sports Shop sold 10 balls, 3 bats, and 2 bases for $99 on Monday. On Tuesday they sold 4 balls, 8 bats, and 2 bases for $78. On Wednesday they sold 2 balls, 3 bats, and 1 base for $33.60. What are the prices of 1 ball, 1 bat, and 1 base?**

First define the variables.

x = price of 1 ball
y = price of 1 bat
z = price of 1 base

Translate the information in the problem into three equations.

$$\begin{aligned} 10x + 3y + 2z &= 99 \\ 4x + 8y + 2z &= 78 \\ 2x + 3y + z &= 33.60 \end{aligned}$$

Subtract the second equation from the first equation to eliminate z.

$$\begin{aligned} 10x + 3y + 2z &= 99 \\ (-)\ 4x + 8y + 2z &= 78 \\ \hline 6x - 5y &= 21 \end{aligned}$$

Multiply the third equation by 2 and subtract from the second equation.

$$\begin{aligned} 4x + 8y + 2z &= 78 \\ (-)\ 4x + 6y + 2z &= 67.20 \\ \hline 2y &= 10.80 \\ y &= 5.40 \end{aligned}$$

Substitute 5.40 for y in the equation $6x - 5y = 21$.

$$\begin{aligned} 6x - 5(5.40) &= 21 \\ 6x &= 48 \\ x &= 8 \end{aligned}$$

Substitute 8 for x and 5.40 for y in one of the original equations to solve for z.

$$\begin{aligned} 10x + 3y + 2z &= 99 \\ 10(8) + 3(5.40) + 2z &= 99 \\ 80 + 16.20 + 2z &= 99 \\ 2z &= 2.80 \\ z &= 1.40 \end{aligned}$$

So a ball costs $8, a bat $5.40, and a base $1.40.

Exercises

1. **FITNESS TRAINING** Carly is training for a triathlon. In her training routine each week, she runs 7 times as far as she swims, and she bikes 3 times as far as she runs. One week she trained a total of 232 miles. How far did she run that week? **56 miles**

2. **ENTERTAINMENT** At the arcade, Ryan, Sara, and Tim played video racing games, pinball, and air hockey. Ryan spent $6 for 6 racing games, 2 pinball games, and 1 game of air hockey. Sara spent $12 for 3 racing games, 4 pinball games, and 5 games of air hockey. Tim spent $12.25 for 2 racing games, 7 pinball games, and 4 games of air hockey. How much did each of the games cost? **Racing game: $0.50; pinball: $0.75; air hockey: $1.50**

3. **FOOD** A natural food store makes its own brand of trail mix out of dried apples, raisins, and peanuts. One pound of the mixture costs $3.18. It contains twice as much peanuts by weight as apples. One pound of dried apples costs $4.48, a pound of raisins $2.40, and a pound of peanuts $3.44. How many ounces of each ingredient are contained in 1 pound of the trail mix? **3 oz of apples, 7 oz of raisins, 6 oz of peanuts**