### P2A.1.2 Systems of Linear Equations Vocabulary

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
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<tbody>
<tr>
<td>System</td>
<td>A set of two or more equations that form a solution point or area</td>
</tr>
<tr>
<td>Linear Function</td>
<td>Each term has an exponent of one and the graphing of the equation results in a straight line</td>
</tr>
<tr>
<td>Solution</td>
<td>The value that when substituted for the variable in a given equation/expression produces a true statement</td>
</tr>
<tr>
<td>Elimination (aka Gaussian and Back Substitution)</td>
<td>A process used to solve systems of equations by combining two equations in a way that cancels a variable</td>
</tr>
<tr>
<td>Substitution</td>
<td>A process used to solve a system of equations by replacing a variable in one equation with an equivalent expression from the other equation</td>
</tr>
<tr>
<td>Matrix (Matrices)</td>
<td>A rectangular array of quantities organized by rows and columns</td>
</tr>
<tr>
<td>Rows</td>
<td>The horizontal in a matrix</td>
</tr>
<tr>
<td>Columns</td>
<td>The vertical in a matrix</td>
</tr>
<tr>
<td>Inverse of a Matrix</td>
<td>The matrix must be square in order to have an inverse; inverse is denoted as $A^{-1}$</td>
</tr>
<tr>
<td>Ordered Triple</td>
<td>The solution of a linear equation of 3 variables</td>
</tr>
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</table>
Graph Systems of Equations  A system of equations is a set of two or more equations containing the same variables. You can solve a system of linear equations by graphing the equations on the same coordinate plane. If the lines intersect, the solution is that intersection point.

**Example** Solve the system of equations by graphing.  \[ x - 2y = 4 \]
\[ x + y = -2 \]

Write each equation in slope-intercept form.
\[ x - 2y = 4 \quad \rightarrow \quad y = \frac{x}{2} - 2 \]
\[ x + y = -2 \quad \rightarrow \quad y = -x - 2 \]

The graphs appear to intersect at \((0, -2)\).

**CHECK** Substitute the coordinates into each equation.
\[ x - 2y = 4 \]
\[ 0 - 2(-2) = 4 \quad \checkmark \]
\[ x + y = -2 \]
\[ 0 + (-2) = -2 \quad \checkmark \]

The solution of the system is \((0, -2)\).

**Exercises**

Solve each system of equations by graphing.

1. \[ y = -\frac{x}{3} + 1 \quad y = \frac{x}{2} - 4 \]
2. \[ y = 2x - 2 \quad y = -x + 4 \]
3. \[ y = -\frac{x}{2} + 3 \quad y = \frac{x}{4} \]
4. \[ 3x - y = 0 \quad x - y = -2 \]
5. \[ 2x + \frac{y}{3} = -7 \quad \frac{x}{2} + y = 1 \]
6. \[ \frac{x}{2} - y = 2 \quad 2x - y = -1 \]
Study Guide and Intervention (continued)

Solving Systems of Equations by Graphing

Classify Systems of Equations The following chart summarizes the possibilities for graphs of two linear equations in two variables.

<table>
<thead>
<tr>
<th>Graphs of Equations</th>
<th>Slopes of Lines</th>
<th>Classification of System</th>
<th>Number of Solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lines intersect</td>
<td>Different slopes</td>
<td>Consistent and independent</td>
<td>One</td>
</tr>
<tr>
<td>Lines coincide (same line)</td>
<td>Same slope, same y-intercept</td>
<td>Consistent and dependent</td>
<td>Infinitely many</td>
</tr>
<tr>
<td>Lines are parallel</td>
<td>Same slope, different y-intercepts</td>
<td>Inconsistent</td>
<td>None</td>
</tr>
</tbody>
</table>

**Example** Graph the system of equations and describe it as consistent and independent, consistent and dependent, or inconsistent.

\[ x - 3y = 6 \]
\[ 2x - y = -3 \]

Write each equation in slope-intercept form.

\[ x - 3y = 6 \quad \rightarrow \quad y = \frac{1}{3}x - 2 \]
\[ 2x - y = -3 \quad \rightarrow \quad y = 2x + 3 \]

The graphs intersect at \((-3, -3)\). Since there is one solution, the system is consistent and independent.

**Exercises** Graph the system of equations and describe it as consistent and independent, consistent and dependent, or inconsistent.

1. \[ 3x + y = -2 \quad \quad \quad \quad 6x + 2y = 10 \]
2. \[ x + 2y = 5 \quad \quad \quad \quad 3x - 15 = -6y \]
3. \[ 2x - 3y = 0 \quad \quad \quad \quad 4x - 6y = 3 \]

4. \[ 2x - y = 3 \quad \quad \quad \quad x + 2y = 4 \]
5. \[ 4x + y = -2 \quad \quad \quad \quad 2x + \frac{y}{2} = -1 \]
6. \[ 3x - y = 2 \quad \quad \quad \quad x + y = 6 \]
**3-2 Study Guide**

**Solving Systems of Equations Algebraically**

**Substitution** To solve a system of linear equations by substitution, first solve for one variable in terms of the other in one of the equations. Then substitute this expression into the other equation and simplify.

**Example** Use substitution to solve the system of equations. \[ 2x - y = 9 \]
\[ x + 3y = -6 \]

Solve the first equation for \( y \) in terms of \( x \).

\[
\begin{align*}
2x - y &= 9 \\
-y &= -2x + 9 \\
y &= 2x - 9
\end{align*}
\]

First equation

Subtract 2x from both sides.

Multiply both sides by \(-1\).

Substitute the expression \( 2x - 9 \) for \( y \) into the second equation and solve for \( x \).

\[
\begin{align*}
x + 3y &= -6 \\
x + 3(2x - 9) &= -6 \\
x + 6x - 27 &= -6 \\
7x - 27 &= -6 \\
7x &= 21 \\
x &= 3
\end{align*}
\]

Second equation

Distributive Property

Add 27 to each side.

Divide each side by 7.

Now, substitute the value 3 for \( x \) in either original equation and solve for \( y \).

\[
\begin{align*}
2x - y &= 9 \\
2(3) - y &= 9
\end{align*}
\]

Replace \( x \) with 3.

Simplify.

Subtract 6 from each side.

Multiply each side by \(-1\).

The solution of the system is \((3, -3)\).

**Exercises**

Solve each system of equations by using substitution.

1. \( 3x + y = 7 \)
   \( 4x + 2y = 16 \)

2. \( 2x + y = 5 \)
   \( 3x - 3y = 3 \)

3. \( 2x + 3y = -3 \)
   \( x + 2y = 2 \)

4. \( 2x - y = 7 \)
   \( 6x - 3y = 14 \)

5. \( 4x - 3y = 4 \)
   \( 2x + y = -8 \)

6. \( 5x + y = 6 \)
   \( 3 - x = 0 \)

7. \( x + 8y = -2 \)
   \( x - 3y = 20 \)

8. \( 2x - y = -4 \)
   \( 4x + y = 1 \)

9. \( x - y = -2 \)
   \( 2x - 3y = 2 \)

10. \( x - 4y = 4 \)
    \( 2x + 12y = 13 \)

11. \( x + 3y = 2 \)
    \( 4x + 12y = 8 \)

12. \( 2x + 2y = 4 \)
    \( x - 2y = 0 \)
Solving Systems of Equations Algebraically

Elimination  To solve a system of linear equations by elimination, add or subtract the equations to eliminate one of the variables. You may first need to multiply one or both of the equations by a constant so that one of the variables has the opposite coefficient in one equation as it has in the other.

Example 1  Use the elimination method to solve the system of equations.

\[ \begin{align*}
2x - 4y &= -26 \\
3x - y &= -24
\end{align*} \]

Multiply the second equation by \(-4\). Then add the equations to eliminate the \(y\) variable.

\[ \begin{align*}
2x - 4y &= -26 \\
3x - y &= -24 \\
\text{Multiply by } -4, \quad -12x + 4y &= 96 \\
\hline
-10x &= 70 \\
x &= -7
\end{align*} \]

Replace \(x\) with \(-7\) and solve for \(y\).

\[ \begin{align*}
2x - 4y &= -26 \\
2(-7) - 4y &= -26 \\
-14 - 4y &= -26 \\
-4y &= -12 \\
y &= 3
\end{align*} \]

The solution is \((-7, 3)\).

Example 2  Use the elimination method to solve the system of equations.

\[ \begin{align*}
3x - 2y &= 4 \\
5x + 3y &= -25
\end{align*} \]

Multiply the first equation by \(3\) and the second equation by \(2\). Then add the equations to eliminate the \(y\) variable.

\[ \begin{align*}
3x - 2y &= 4 \\
5x + 3y &= -25 \\
\text{Multiply by } 3, \quad 9x - 6y &= 12 \\
\text{Multiply by } 2, \quad 10x + 6y &= -50 \\
\hline
19x &= -38 \\
x &= -2
\end{align*} \]

Replace \(x\) with \(-2\) and solve for \(y\).

\[ \begin{align*}
3x - 2y &= 4 \\
3(-2) - 2y &= 4 \\
-6 - 2y &= 4 \\
-2y &= 10 \\
y &= -5
\end{align*} \]

The solution is \((-2, -5)\).

Exercises

Solve each system of equations by using elimination.

1. \[ \begin{align*}
2x - y &= 7 \\
3x + y &= 8
\end{align*} \]

2. \[ \begin{align*}
x - 2y &= 4 \\
-x + 6y &= 12
\end{align*} \]

3. \[ \begin{align*}
3x + 4y &= -10 \\
x - 4y &= 2
\end{align*} \]

4. \[ \begin{align*}
3x - y &= 12 \\
5x + 2y &= 20
\end{align*} \]

5. \[ \begin{align*}
4x - y &= 6 \\
2x - \frac{y}{2} &= 4
\end{align*} \]

6. \[ \begin{align*}
5x + 2y &= 12 \\
-6x - 2y &= -14
\end{align*} \]

7. \[ \begin{align*}
2x + y &= 8 \\
3x + \frac{3}{2}y &= 12
\end{align*} \]

8. \[ \begin{align*}
7x + 2y &= -1 \\
4x - 3y &= -13
\end{align*} \]

9. \[ \begin{align*}
3x + 8y &= -6 \\
x - y &= 9
\end{align*} \]

10. \[ \begin{align*}
5x + 4y &= 12 \\
7x - 6y &= 40
\end{align*} \]

11. \[ \begin{align*}
-4x + y &= -12 \\
4x + 2y &= 6
\end{align*} \]

12. \[ \begin{align*}
5x + 2y &= -8 \\
4x + 3y &= 2
\end{align*} \]
Study Guide and Intervention
Solving Systems of Equations in Three Variables

Systems in Three Variables  Use the methods used for solving systems of linear equations in two variables to solve systems of equations in three variables. A system of three equations in three variables can have a unique solution, infinitely many solutions, or no solution. A solution is an ordered triple.

Example  Solve this system of equations.  \[3x + y - z = -6\]
\[2x - y + 2z = 8\]
\[4x + y - 3z = -21\]

Step 1  Use elimination to make a system of two equations in two variables.

\[
\begin{align*}
3x + y - z &= -6 \\
(+) 2x - y + 2z &= 8
\end{align*}
\]

First equation

\[
\begin{align*}
2x - y + 2z &= 8 \\
(+) 4x + y - 3z &= -21
\end{align*}
\]

Second equation

Third equation

\[
\begin{align*}
5x + z &= 2 \\
Add to eliminate y.
\end{align*}
\]

Step 2  Solve the system of two equations.

\[
\begin{align*}
5x + z &= 2 \\
(+) 6x - z &= -13
\end{align*}
\]

Add to eliminate z.

\[
\begin{align*}
11x &= -11 \\
11 &= -11
\end{align*}
\]

Divide both sides by 11.

\[
\begin{align*}
x &= -1
\end{align*}
\]

Substitute \(-1\) for \(x\) in one of the equations with two variables and solve for \(z\).

\[
\begin{align*}
5x + z &= 2 \\
5(-1) + z &= 2 \\
-5 + z &= 2 \\
-5 + z &= 2 \\
\end{align*}
\]

Replace \(x\) with \(-1\).

\[
\begin{align*}
z &= 7
\end{align*}
\]

Add 5 to both sides.

The result so far is \(x = -1\) and \(z = 7\).

Step 3  Substitute \(-1\) for \(x\) and 7 for \(z\) in one of the original equations with three variables.

\[
\begin{align*}
3x + y - z &= -6 \\
3(-1) + y - 7 &= -6 \\
-3 + y - 7 &= -6 \\
y &= 4
\end{align*}
\]

Replace \(x\) with \(-1\) and \(z\) with 7.

\[
\begin{align*}
\text{Multiply.}
y &= 4
\end{align*}
\]

The solution is \((-1, 4, 7)\).

Exercises

Solve each system of equations.

1. \(2x + 3y - z = 0\)
   \(x - 2y - 4z = 14\)
   \(3x + y - 8z = 17\)

2. \(2x - y + 4z = 11\)
   \(x + 2y - 6z = -11\)
   \(3x - 2y - 10z = 11\)

3. \(x - 2y + z = 8\)
   \(2x + y - z = 0\)
   \(3x - 6y + 3z = 24\)

4. \(3x - y - z = 5\)
   \(3x + 2y - z = 11\)
   \(6x - 3y + 2z = -12\)

5. \(2x - 4y - z = 10\)
   \(4x - 8y - 2z = 16\)
   \(3x + y + z = 12\)

6. \(x - 6y + 4z = 2\)
   \(2x + 4y - 8z = 16\)
   \(x - 2y = 5\)
Real-World Problems

Example

The Laredo Sports Shop sold 10 balls, 3 bats, and 2 bases for $99 on Monday. On Tuesday they sold 4 balls, 8 bats, and 2 bases for $78. On Wednesday they sold 2 balls, 3 bats, and 1 base for $33.60. What are the prices of 1 ball, 1 bat, and 1 base?

First define the variables.

\[ x = \text{price of 1 ball} \]
\[ y = \text{price of 1 bat} \]
\[ z = \text{price of 1 base} \]

Translate the information in the problem into three equations.

\[ 10x + 3y + 2z = 99 \]
\[ 4x + 8y + 2z = 78 \]
\[ 2x + 3y + z = 33.60 \]

Subtract the second equation from the first equation to eliminate \( z \).

\[
\begin{align*}
10x + 3y + 2z &= 99 \\
(\text{\(-)}) 4x + 8y + 2z &= 78 \\
6x - 5y &= 21
\end{align*}
\]

Multiply the third equation by 2 and subtract from the second equation.

\[
\begin{align*}
4x + 8y + 2z &= 78 \\
(-) 4x + 6y + 2z &= 67.20 \\
2y &= 10.80 \\
y &= 5.40
\end{align*}
\]

Substitute 5.40 for \( y \) in the equation

\[
6x - 5(5.40) = 21
\]

\[
6x = 48
\]

\[
x = 8
\]

Substitute 8 for \( x \) and 5.40 for \( y \) in one of the original equations to solve for \( z \).

\[
10x + 3y + 2z = 99
\]

\[
10(8) + 3(5.40) + 2z = 99
\]

\[
80 + 16.20 + 2z = 99
\]

\[
2z = 2.80
\]

\[
z = 1.40
\]

So a ball costs $8, a bat $5.40, and a base $1.40.

Exercises

1. **FITNESS TRAINING**  Carly is training for a triathlon. In her training routine each week, she runs 7 times as far as she swims, and she bikes 3 times as far as she runs. One week she trained a total of 232 miles. How far did she run that week?

2. **ENTERTAINMENT**  At the arcade, Ryan, Sara, and Tim played video racing games, pinball, and air hockey. Ryan spent $6 for 6 racing games, 2 pinball games, and 1 game of air hockey. Sara spent $12 for 3 racing games, 4 pinball games, and 5 games of air hockey. Tim spent $12.25 for 2 racing games, 7 pinball games, and 4 games of air hockey. How much did each of the games cost?

3. **FOOD**  A natural food store makes its own brand of trail mix out of dried apples, raisins, and peanuts. One pound of the mixture costs $3.18. It contains twice as much peanuts by weight as apples. One pound of dried apples costs $4.48, a pound of raisins $2.40, and a pound of peanuts $3.44. How many ounces of each ingredient are contained in 1 pound of the trail mix?