

4-1 Skills Practice**Introduction to Matrices**

State the dimensions of each matrix.

1. $\begin{bmatrix} 3 & 2 & 4 \\ -1 & 4 & 0 \end{bmatrix}$

2. $[0 \ 15]$

3. $\begin{bmatrix} 3 & 2 \\ 1 & 8 \end{bmatrix}$

4. $\begin{bmatrix} 6 & 1 & 2 \\ -3 & 4 & 5 \\ -2 & 7 & 9 \end{bmatrix}$

5. $\begin{bmatrix} 9 & 3 & -3 & -6 \\ 3 & 4 & -4 & 5 \end{bmatrix}$

6. $\begin{bmatrix} -1 \\ -1 \\ -1 \\ -3 \end{bmatrix}$

Identify each element for the following matrices.

$$A = \begin{bmatrix} 9 & 6 & 7 \\ 2 & 5 & 0 \\ 10 & 3 & 11 \end{bmatrix},$$

$$B = \begin{bmatrix} 5 & -2 & 4 & 3 \\ 0 & 8 & 12 & -1 \end{bmatrix},$$

$$C = \begin{bmatrix} 8 & 1 & 6 \\ 7 & 0 & 2 \\ 4 & 9 & 5 \\ 3 & 12 & 10 \end{bmatrix}.$$

7. b_{22}

8. c_{42}

9. b_{11}

10. a_{33}

11. c_{14}

12. a_{21}

13. c_{33}

14. b_{13}

15. a_{12}

4-1 Practice

Introduction to Matrices

State the dimensions of each matrix.

1. $[-3 \ -3 \ 7]$ 2. $\begin{bmatrix} 5 & 8 & -1 \\ -2 & 1 & 8 \end{bmatrix}$ 3. $\begin{bmatrix} -2 & 2 & -2 & 3 \\ 5 & 16 & 0 & 0 \\ 4 & 7 & -1 & 4 \end{bmatrix}$

Identify each element for the following matrices.

$$A = \begin{bmatrix} 4 & 7 & 0 \\ 9 & 8 & -4 \\ 3 & 0 & 5 \\ -1 & 2 & 6 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 6 & -1 & 0 \\ 9 & 5 & 7 & 2 \end{bmatrix}.$$

4. b_{23} 5. a_{42} 6. b_{11}
7. a_{32} 8. b_{14} 9. a_{23}

10. TICKET PRICES The table at the right gives ticket prices for a concert. Write a 2×3 matrix that represents the cost of a ticket.

	Child	Student	Adult
Cost Purchased in Advance	\$6	\$12	\$18
Cost Purchased at the Door	\$8	\$15	\$22

11. CONSTRUCTION During each of the last three weeks, a road-building crew has used three truckloads of gravel. The table at the right shows the amount of gravel in each load.

	Week 1	Week 2	Week 3
Load 1	40 tons	40 tons	32 tons
Load 2	32 tons	40 tons	24 tons
Load 3	24 tons	32 tons	24 tons

- a. Write a matrix for the amount of gravel in each load.
- b. What are the dimensions of the matrix?

4-2 Skills Practice**Operations with Matrices**

Perform the indicated operations. If the matrix does not exist, write *impossible*.

1. $[5 \ -4] + [4 \ 5]$

2. $\begin{bmatrix} 8 & 3 \\ -1 & -1 \end{bmatrix} - \begin{bmatrix} 0 & -7 \\ 6 & 2 \end{bmatrix}$

3. $[3 \ 1 \ 6] + \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix}$

4. $\begin{bmatrix} 5 & -1 & 2 \\ 1 & 8 & -6 \end{bmatrix} + \begin{bmatrix} 9 & 9 & 2 \\ 4 & 6 & 4 \end{bmatrix}$

5. $3[9 \ 4 \ -3]$

6. $[6 \ -3] - 4[4 \ 7]$

7. $-2 \begin{bmatrix} -2 & 5 \\ 5 & 9 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

8. $3 \begin{bmatrix} 8 \\ 0 \\ -3 \end{bmatrix} - 4 \begin{bmatrix} 2 \\ 2 \\ 10 \end{bmatrix}$

9. $5 \begin{bmatrix} -4 & 6 \\ 10 & 1 \\ -1 & 1 \end{bmatrix} + 2 \begin{bmatrix} 6 & 5 \\ -3 & -2 \\ 1 & 0 \end{bmatrix}$

10. $3 \begin{bmatrix} 3 & 1 & 3 \\ -4 & 7 & 5 \end{bmatrix} - 2 \begin{bmatrix} 1 & -1 & 5 \\ 6 & 6 & -3 \end{bmatrix}$

Use matrices A , B , and C to find the following.

$$A = \begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}, B = \begin{bmatrix} 2 & 2 \\ 1 & -2 \end{bmatrix}, \text{ and } C = \begin{bmatrix} -3 & 4 \\ 3 & 1 \end{bmatrix}.$$

11. $A + B$

12. $B - C$

13. $B - A$

14. $A + B + C$

15. $3B$

16. $-5C$

17. $A - 4C$

18. $2B + 3A$

4-2 Practice

Operations with Matrices

Perform the indicated operations. If the matrix does not exist, write *impossible*.

1. $\begin{bmatrix} 2 & -1 \\ 3 & 7 \\ 14 & -9 \end{bmatrix} + \begin{bmatrix} -6 & 9 \\ 7 & -11 \\ -8 & 17 \end{bmatrix}$

2. $\begin{bmatrix} 4 \\ -71 \\ 18 \end{bmatrix} - \begin{bmatrix} -67 \\ 45 \\ -24 \end{bmatrix}$

3. $-3 \begin{bmatrix} -1 & 0 \\ 17 & -11 \end{bmatrix} + 4 \begin{bmatrix} -3 & 16 \\ -21 & 12 \end{bmatrix}$

4. $7 \begin{bmatrix} 2 & -1 & 8 \\ 4 & 7 & 9 \end{bmatrix} - 2 \begin{bmatrix} -1 & 4 & -3 \\ 7 & 2 & -6 \end{bmatrix}$

5. $-2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + 4 \begin{bmatrix} 0 \\ 5 \end{bmatrix} - \begin{bmatrix} 10 \\ 18 \end{bmatrix}$

6. $\frac{3}{4} \begin{bmatrix} 8 & 12 \\ -16 & 20 \end{bmatrix} + \frac{2}{3} \begin{bmatrix} 27 & -9 \\ 54 & -18 \end{bmatrix}$

Use matrices $A = \begin{bmatrix} 4 & -1 & 0 \\ -3 & 6 & 2 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 4 & 5 \\ 1 & 0 & 9 \end{bmatrix}$, and $C = \begin{bmatrix} 10 & -8 & 6 \\ -6 & -4 & 20 \end{bmatrix}$ to find the following.

7. $A - B$

8. $A - C$

9. $-3B$

10. $4B - A$

11. $-2B - 3C$

12. $A + 0.5C$

13. ECONOMICS Use the table that shows loans by an economic development board to women and men starting new businesses.

	Women		Men	
	Businesses	Loan Amount (\$)	Businesses	Loan Amount (\$)
2003	27	\$567,000	36	\$864,000
2004	41	\$902,000	32	\$672,000
2005	35	\$777,000	28	\$562,000

a. Write two matrices that represent the number of new businesses and loan amounts, one for women and one for men.

b. Find the sum of the numbers of new businesses and loan amounts for both men and women over the three-year period expressed as a matrix.

14. PET NUTRITION Use the table that gives nutritional information for two types of dog food. Find the difference in the percent of protein, fat, and fiber between Mix B and Mix A expressed as a matrix.

	% Protein	% Fat	% Fiber
Mix A	22	12	5
Mix B	24	8	8

4-3 Skills Practice**Multiplying Matrices**

Determine whether each matrix product is defined. If so, state the dimensions of the product.

1. $A_{2 \times 5} \cdot B_{5 \times 1}$

2. $M_{1 \times 3} \cdot N_{3 \times 2}$

3. $B_{3 \times 2} \cdot A_{3 \times 2}$

4. $R_{4 \times 4} \cdot S_{4 \times 1}$

5. $X_{3 \times 3} \cdot Y_{3 \times 4}$

6. $A_{6 \times 4} \cdot B_{4 \times 5}$

Find each product, if possible.

7. $\begin{bmatrix} 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

8. $\begin{bmatrix} 5 & 6 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & -5 \\ 3 & 1 \end{bmatrix}$

9. $\begin{bmatrix} 1 & 3 \\ -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ -2 \end{bmatrix}$

10. $\begin{bmatrix} 3 \\ -2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 \\ -1 & 1 \end{bmatrix}$

11. $\begin{bmatrix} -3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 0 & -1 \\ 2 & 2 \end{bmatrix}$

12. $\begin{bmatrix} -1 \\ 3 \end{bmatrix} \cdot [2 \ -3 \ -2]$

13. $\begin{bmatrix} 5 \\ 6 \\ -3 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 8 \end{bmatrix}$

14. $\begin{bmatrix} 2 & -2 \\ 4 & 5 \\ -3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 3 \\ 3 & 0 \end{bmatrix}$

15. $\begin{bmatrix} -4 & 4 \\ -2 & 1 \\ 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 3 & -3 \\ 0 & 2 \end{bmatrix}$

16. $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$

Use $A = \begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 2 \\ 5 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 3 & -1 \\ 1 & 0 \end{bmatrix}$, and $c = 2$ to determine whether the following equations are true for the given matrices.

17. $c(AC) = A(cC)$

18. $AB = BA$

19. $B(A + C) = AB + BC$

20. $c(A - B) = cA - cB$

4-3 Practice

Multiplying Matrices

Determine whether each matrix product is defined. If so, state the dimensions of the product.

1. $A_{7 \times 4} \cdot B_{4 \times 3}$

2. $A_{3 \times 5} \cdot M_{5 \times 8}$

3. $M_{2 \times 1} \cdot A_{1 \times 6}$

4. $M_{3 \times 2} \cdot A_{3 \times 2}$

5. $P_{1 \times 9} \cdot Q_{9 \times 1}$

6. $P_{9 \times 1} \cdot Q_{1 \times 9}$

Find each product, if possible.

7. $\begin{bmatrix} 2 & 4 \\ 3 & -1 \end{bmatrix} \cdot \begin{bmatrix} 3 & -2 & 7 \\ 6 & 0 & -5 \end{bmatrix}$

8. $\begin{bmatrix} 2 & 4 \\ 7 & -1 \end{bmatrix} \cdot \begin{bmatrix} -3 & 0 \\ 2 & 5 \end{bmatrix}$

9. $\begin{bmatrix} -3 & 0 \\ 2 & 5 \end{bmatrix} \cdot \begin{bmatrix} 2 & 4 \\ 7 & -1 \end{bmatrix}$

10. $\begin{bmatrix} 3 & -2 & 7 \\ 6 & 0 & -5 \end{bmatrix} \cdot \begin{bmatrix} 3 & -2 & 7 \\ 6 & 0 & -5 \end{bmatrix}$

11. $\begin{bmatrix} 4 & 0 & 2 \\ & & 1 \\ & & 3 \\ & & -1 \end{bmatrix}$

12. $\begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 4 & 0 & 2 \end{bmatrix}$

13. $\begin{bmatrix} -6 & 2 \\ 3 & -1 \end{bmatrix} \cdot \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

14. $\begin{bmatrix} -15 & -9 \end{bmatrix} \cdot \begin{bmatrix} 6 & 11 \\ 23 & -10 \end{bmatrix}$

Use $A = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 0 \\ -2 & -1 \end{bmatrix}$, $C = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$, and $c = 3$ to determine whether the following equations are true for the given matrices.

15. $AC = CA$

16. $A(B + C) = BA + CA$

17. $(AB)c = c(AB)$

18. $(A + C)B = B(A + C)$

19. RENTALS For their one-week vacation, the Montoyas can rent a 2-bedroom condominium for \$1796, a 3-bedroom condominium for \$2165, or a 4-bedroom condominium for \$2538. The table shows the number of units in each of three complexes.

	2-Bedroom	3-Bedroom	4-Bedroom
Sun Haven	36	24	22
Surfside	29	32	42
Seabreeze	18	22	18

a. Write a matrix that represents the number of each type of unit available at each complex and a matrix that represents the weekly charge for each type of unit.

b. If all of the units in the three complexes are rented for the week at the rates given the Montoyas, express the income of each of the three complexes as a matrix.

c. What is the total income of all three complexes for the week?

4-5 Skills Practice**Determinants and Cramer's Rule****Evaluate each determinant.**

1. $\begin{vmatrix} 5 & 2 \\ 1 & 3 \end{vmatrix}$

2. $\begin{vmatrix} 10 & 9 \\ 5 & 8 \end{vmatrix}$

3. $\begin{vmatrix} 1 & 6 \\ 1 & 7 \end{vmatrix}$

4. $\begin{vmatrix} 2 & 5 \\ 3 & 1 \end{vmatrix}$

5. $\begin{vmatrix} 0 & 9 \\ 5 & 8 \end{vmatrix}$

6. $\begin{vmatrix} 3 & 12 \\ 2 & 8 \end{vmatrix}$

7. $\begin{vmatrix} -5 & 2 \\ 8 & -6 \end{vmatrix}$

8. $\begin{vmatrix} -3 & 1 \\ 8 & -7 \end{vmatrix}$

9. $\begin{vmatrix} 9 & -2 \\ -4 & 1 \end{vmatrix}$

10. $\begin{vmatrix} 1 & -5 \\ 1 & 6 \end{vmatrix}$

11. $\begin{vmatrix} 1 & -3 \\ -3 & 4 \end{vmatrix}$

12. $\begin{vmatrix} -12 & 4 \\ 1 & 4 \end{vmatrix}$

13. $\begin{vmatrix} 3 & -5 \\ 6 & -11 \end{vmatrix}$

14. $\begin{vmatrix} -1 & -3 \\ 5 & -2 \end{vmatrix}$

15. $\begin{vmatrix} -1 & -14 \\ 5 & 2 \end{vmatrix}$

16. $\begin{vmatrix} -1 & 2 \\ 0 & 4 \end{vmatrix}$

17. $\begin{vmatrix} 2 & 2 \\ -1 & 4 \end{vmatrix}$

18. $\begin{vmatrix} -1 & 6 \\ 2 & 5 \end{vmatrix}$

Evaluate each determinant using diagonals.

19. $\begin{vmatrix} 2 & -1 & 1 \\ 3 & 2 & -1 \\ 2 & 3 & -2 \end{vmatrix}$

20. $\begin{vmatrix} 6 & -1 & 1 \\ 5 & 2 & -1 \\ 1 & 3 & -2 \end{vmatrix}$

21. $\begin{vmatrix} 2 & 6 & 1 \\ 3 & 5 & -1 \\ 2 & 1 & -2 \end{vmatrix}$

22. $\begin{vmatrix} 2 & -1 & 6 \\ 3 & 2 & 5 \\ 2 & 3 & 1 \end{vmatrix}$

23. $\begin{vmatrix} 3 & -1 & 2 \\ 1 & 0 & 4 \\ 3 & -2 & 0 \end{vmatrix}$

24. $\begin{vmatrix} 3 & 2 & 2 \\ 1 & -1 & 4 \\ 3 & -1 & 0 \end{vmatrix}$

4-5 Practice**Determinants and Cramer's Rule**

Evaluate each determinant.

1. $\begin{vmatrix} 1 & 6 \\ 2 & 7 \end{vmatrix}$

2. $\begin{vmatrix} 9 & 6 \\ 3 & 2 \end{vmatrix}$

3. $\begin{vmatrix} 4 & 1 \\ -2 & -5 \end{vmatrix}$

4. $\begin{vmatrix} -14 & -3 \\ 2 & -2 \end{vmatrix}$

5. $\begin{vmatrix} 4 & -3 \\ -12 & 4 \end{vmatrix}$

6. $\begin{vmatrix} 2 & -5 \\ 5 & -11 \end{vmatrix}$

7. $\begin{vmatrix} 3 & -4 \\ 3.75 & 5 \end{vmatrix}$

8. $\begin{vmatrix} 2 & -1 \\ 3 & -9.5 \end{vmatrix}$

9. $\begin{vmatrix} 0.5 & -0.7 \\ 0.4 & -0.3 \end{vmatrix}$

Evaluate each determinant using expansion by diagonals.

10. $\begin{vmatrix} -2 & 3 & 1 \\ 0 & 4 & -3 \\ 2 & 5 & -1 \end{vmatrix}$

11. $\begin{vmatrix} 2 & -4 & 1 \\ 3 & 0 & 9 \\ -1 & 5 & 7 \end{vmatrix}$

12. $\begin{vmatrix} 2 & 1 & 1 \\ 1 & -1 & -2 \\ 1 & 1 & -1 \end{vmatrix}$

13. $\begin{vmatrix} 0 & -4 & 0 \\ 2 & -1 & 1 \\ 3 & -2 & 5 \end{vmatrix}$

14. $\begin{vmatrix} 2 & 7 & -6 \\ 8 & 4 & 0 \\ 1 & -1 & 3 \end{vmatrix}$

15. $\begin{vmatrix} -12 & 0 & 3 \\ 7 & 5 & -1 \\ 4 & 2 & -6 \end{vmatrix}$

Use Cramer's Rule to solve each system of equation.

16. $4x - 2y = -4$

$3x + y = 18$

17. $5x + 4y = 10$

$-3x - 2y = -8$

18. $-2x - 3y = -14$

$4x - y = 0$

19. $6x + 6y = 9$

$4x - 4y = -42$

20. $5x - 6 = 3y$

$5y = 54 + 3x$

21. $\frac{x}{2} + \frac{y}{4} = 2$

$\frac{x}{4} - \frac{y}{6} = -6$

25. GEOMETRY Find the area of a triangle whose vertices have coordinates (3, 5), (6, -5), and (-4, 10).

26. LAND MANAGEMENT A fish and wildlife management organization uses a GIS (geographic information system) to store and analyze data for the parcels of land it manages. All of the parcels are mapped on a grid in which 1 unit represents 1 acre. If the coordinates of the corners of a parcel are (-8, 10), (6, 17), and (2, -4), how many acres is the parcel?

4-6 Skills Practice**Inverse Matrices and Systems of Equations****Determine whether the matrices in each pair are inverses.**

1. $X = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, Y = \begin{bmatrix} -1 & 0 \\ 1 & 1 \end{bmatrix}$

2. $P = \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}, Q = \begin{bmatrix} -1 & 3 \\ 1 & -2 \end{bmatrix}$

3. $M = \begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix}, N = \begin{bmatrix} -1 & 0 \\ 0 & -3 \end{bmatrix}$

4. $A = \begin{bmatrix} -2 & 5 \\ -1 & 2 \end{bmatrix}, B = \begin{bmatrix} 2 & -5 \\ 1 & -2 \end{bmatrix}$

5. $V = \begin{bmatrix} 0 & 7 \\ -7 & 0 \end{bmatrix}, W = \begin{bmatrix} 0 & -\frac{1}{7} \\ \frac{1}{7} & 0 \end{bmatrix}$

6. $X = \begin{bmatrix} -1 & 4 \\ 1 & 2 \end{bmatrix}, Y = \begin{bmatrix} -\frac{1}{3} & \frac{2}{3} \\ \frac{1}{6} & \frac{1}{6} \end{bmatrix}$

7. $G = \begin{bmatrix} 4 & -3 \\ 1 & 2 \end{bmatrix}, H = \begin{bmatrix} \frac{2}{11} & \frac{3}{11} \\ -\frac{1}{11} & \frac{4}{11} \end{bmatrix}$

8. $D = \begin{bmatrix} -4 & -4 \\ -4 & 4 \end{bmatrix}, E = \begin{bmatrix} -0.125 & -0.125 \\ -0.125 & -0.125 \end{bmatrix}$

Find the inverse of each matrix, if it exists.

9. $\begin{bmatrix} 0 & 2 \\ 4 & 0 \end{bmatrix}$

10. $\begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix}$

11. $\begin{bmatrix} 9 & 3 \\ 6 & 2 \end{bmatrix}$

12. $\begin{bmatrix} -2 & -4 \\ 6 & 0 \end{bmatrix}$

13. $\begin{bmatrix} 1 & -1 \\ 3 & 3 \end{bmatrix}$

14. $\begin{bmatrix} 3 & 6 \\ -1 & -2 \end{bmatrix}$

Use a matrix equation to solve each system of equations.

15. $p - 3q = 6$
 $2p + 3q = -6$

16. $-x - 3y = 2$
 $-4x - 5y = 1$

17. $2m + 2n = -8$
 $6m + 4n = -18$

18. $-3a + b = -9$
 $5a - 2b = 14$

4-6 Practice

Inverse Matrices and Systems of Equations

Determine whether each pair of matrices are inverses.

1. $M = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}, N = \begin{bmatrix} -2 & 1 \\ 3 & -2 \end{bmatrix}$

2. $X = \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}, Y = \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$

3. $A = \begin{bmatrix} 3 & 1 \\ -4 & 2 \end{bmatrix}, B = \begin{bmatrix} \frac{1}{5} & -\frac{1}{10} \\ \frac{2}{5} & \frac{3}{10} \end{bmatrix}$

4. $P = \begin{bmatrix} 6 & -2 \\ -2 & 3 \end{bmatrix}, Q = \begin{bmatrix} \frac{3}{14} & \frac{1}{7} \\ \frac{1}{7} & \frac{3}{7} \end{bmatrix}$

Determine whether each statement is *true* or *false*.

- 5. All square matrices have multiplicative inverses.
- 6. All square matrices have multiplicative identities.

Find the inverse of each matrix, if it exists.

7. $\begin{bmatrix} 4 & 5 \\ -4 & -3 \end{bmatrix}$

8. $\begin{bmatrix} 2 & 0 \\ 3 & 5 \end{bmatrix}$

9. $\begin{bmatrix} -1 & 3 \\ 4 & -7 \end{bmatrix}$

10. $\begin{bmatrix} 2 & 5 \\ -1 & 3 \end{bmatrix}$

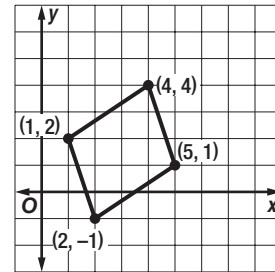
11. $\begin{bmatrix} 2 & -5 \\ 3 & 1 \end{bmatrix}$

12. $\begin{bmatrix} 4 & 6 \\ 6 & 9 \end{bmatrix}$

13. **GEOMETRY** Use the figure at the right.

a. Write the vertex matrix A for the rectangle.

b. Use matrix multiplication to find BA if $B = \begin{bmatrix} 1.5 & 0 \\ 0 & 1.5 \end{bmatrix}$.



c. Graph the vertices of the transformed quadrilateral on the previous graph. Describe the transformation.

d. Make a conjecture about what transformation B^{-1} describes on a coordinate plane.

14. **CODES** Use the alphabet table below and the inverse of coding matrix $C = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ to decode this message:

19 | 14 | 11 | 13 | 11 | 22 | 55 | 65 | 57 | 60 | 2 | 1 | 52 | 47 | 33 | 51 | 56 | 55.

CODE													
A	1	B	2	C	3	D	4	E	5	F	6	G	7
H	8	I	9	J	10	K	11	L	12	M	13	N	14
O	15	P	16	Q	17	R	18	S	19	T	20	U	21
V	22	W	23	X	24	Y	25	Z	26	-	0		

Multivariable Linear Systems and Row Operations

Write the augmented matrix for each system of linear equations.

1) $5x - 2y = -6$
 $-x + 5y = 15$

2) $-3x - 4y = 20$
 $3x - 5y = 25$

3) $x + 3y - 2z = -11$
 $-2x - 5y + 3z = 17$
 $4x - z = 1$

4) $-2x - 4y - 5z = 11$
 $-x + 4z = -25$
 $-3x - 5y + z = -25$

Write the system of linear equations for each augmented matrix.

5) $\left[\begin{array}{cc|c} 3 & 4 & 1 \\ -3 & 2 & 23 \end{array} \right]$

6) $\left[\begin{array}{cc|c} -5 & 1 & -16 \\ 1 & 5 & -2 \end{array} \right]$

7) $\left[\begin{array}{ccc|c} 3 & -1 & 1 & 8 \\ 0 & -1 & 2 & -10 \\ -2 & 2 & 2 & -8 \end{array} \right]$

8) $\left[\begin{array}{ccc|c} -5 & -4 & 3 & -8 \\ 1 & 0 & 4 & 0 \\ 3 & -5 & 5 & -10 \end{array} \right]$

Find the reduced row-echelon form for each system of linear equations.

9) $5x - 4y = -10$
 $-x + y = 2$

10) $4x - 2y = 2$
 $5x - 2y + z = 7$
 $3x + 4y - z = 3$

11) $x - y + 2z = -1$
 $-3x + 3y + 5z = 3$
 $2x - 2y = -2$

12) $3x + 3y = -12$
 $-4x - 2y + 2z = -14$
 $x + 3y + 2z = 11$

Solve each system of linear equations using Gaussian or Gauss-Jordan elimination.

13) $-3x - 4y = -5$
 $4x + 3y = 9$

14) $2x + 5y + z = -12$
 $-x + 4y + 3z = -4$
 $5x - 2z = -13$

15) $3x + 2y - 3z = 13$
 $4x + 4z = 12$
 $-2x - y + z = -8$

16) $-2x - 4y + 4z = 14$
 $4x + 2y + 4z = -4$
 $x + 2z = -2$

3x3 System Applications

Example 1: (this problem is already done for you)

Jesse, Maria and Charles went to the local craft store to purchase supplies for making decorations for the upcoming dance at the high school. Jesse purchased three sheets of craft paper, four boxes of markers and five glue sticks. His bill, before taxes was \$24.40. Maria spent \$30.40 when she bought six sheets of craft paper, five boxes of markers and two glue sticks. Charles, purchases totaled \$13.40 when he bought three sheets of craft paper, two boxes of markers and one glue stick. Determine the unit cost of each item.

Let **p** represent the number of sheets of craft paper.

Let **m** represent the number of boxes of markers.

Let **g** represent the number of glue sticks.

Express the problem as a system of linear equations:

$$3p + 4m + 5g = \$24.40$$

$$6p + 5m + 2g = \$30.40$$

$$3p + 2m + g = \$13.40$$

Rewrite the system of equations as a matrix:

$$\begin{bmatrix} 3 & 4 & 5 & 24.40 \\ 6 & 5 & 2 & 30.40 \\ 3 & 2 & 1 & 13.40 \end{bmatrix}$$

Put the 3 x 4 matrix into your calculator after rref:

$$\begin{aligned} & \text{rref} \left(\begin{bmatrix} 3 & 4 & 5 & 24.40 \\ 6 & 5 & 2 & 30.40 \\ 3 & 2 & 1 & 13.40 \end{bmatrix} \right) \\ & = \begin{bmatrix} 1 & 0 & 0 & 1.75 \\ 0 & 1 & 0 & 3.60 \\ 0 & 0 & 1 & 0.95 \end{bmatrix} \end{aligned}$$

The unit cost of each item is: 1 sheet of craft paper = \$1.75

1 box of markers = \$3.60

1 glue stick = \$0.95

Example 2: (this problem is already done for you)

Rafael, an exchange student from Brazil, made phone calls within Canada, to the United States, and to Brazil. The rates per minute for these calls vary for the different countries. Use the information in the following table to determine the rates.

Month	Time within Canada (min)	Time to the U.S. (min)	Time to Brazil (min)	Charges (\$)
September	90	120	180	\$252.00
October	70	100	120	\$184.00
November	50	110	150	\$206.00

Let c represent the rate for calls within Canada.

Let u represent the rate for calls to the United States.

Let b represent the rate for calls to Brazil.

Express the problem as a system of linear equations:

$$90c + 120u + 180b = \$252.00$$

$$70c + 100u + 120b = \$184.00$$

$$50c + 110u + 150b = \$206.00$$

Rewrite the system of equations as a matrix:

$$\begin{bmatrix} 90 & 120 & 180 & 252 \\ 70 & 100 & 120 & 184 \\ 50 & 110 & 150 & 206 \end{bmatrix}$$

Put the 3 x 4 matrix into your calculator after rref:

$$\begin{aligned} & \text{rref} \left(\begin{bmatrix} 90 & 120 & 180 & 252 \\ 70 & 100 & 120 & 184 \\ 50 & 110 & 150 & 206 \end{bmatrix} \right) \\ &= \begin{bmatrix} 1 & 0 & 0 & 0.40 \\ 0 & 1 & 0 & 0.60 \\ 0 & 0 & 1 & 0.80 \end{bmatrix} \end{aligned}$$

The cost of minutes within Canada is \$0.40/min. The cost of minutes to the United States is \$0.60/min. The cost of minutes to Brazil is \$0.80/min.

Exercises: (you need to do these 4 problems)

1. A local computer company sells three types of laptop computers to three nearby stores. The number of laptops ordered by each store and the amount owing to the company for the order is shown in the following table:

Store	Laptop A	Laptop B	Laptop C	Amount Owing(\$)
Wal-Mart	10	8	6	21 200
Sears	7	9	5	18 700
Target	8	4	3	13 000

Write a system of equations to represent the above information and determine the unit price of each type of laptop computer.

3. Cory, Josh and Dan went shopping for Halloween treats. Cory bought 3 chocolate pumpkins, 4 masks and 8 candy witches. He spent \$36.65. Josh bought 5 chocolate pumpkins, 3 masks and 10 candy witches. He spent \$37.50. Dan bought 4 chocolate pumpkins, 5 masks and 6 candy witches. He spent \$43.45. Write a system of equations to represent this problem and algebraically calculate the unit price of each item purchased.
4. Janet, Larry and Sam bought decorations to decorate the clubhouse for a Christmas party. The number of items bought by each person is given in the table along with the total cost of each purchase. Write a system of equations to represent this problem and algebraically calculate the unit price of each item. Each item was bought at the same store.

Name of the shopper	Number of rolls of garland	Number of wreaths	Number of poinsettias	Cost
Janet	2	4	2	\$49.50
Larry	3	2	4	\$57.75
Sam	3	3	1	\$38.50