Converting Quadratics: Vertex Form to Standard Form

Sketch the graph of each function.

1) \( y = (x - 2)^2 + 3 \)
2) \( y = (x + 1)^2 - 3 \)

3) \( y = -(x + 3)^2 - 3 \)
4) \( y = \frac{1}{2}(x - 2)^2 - 4 \)

5) \( y = -2(x + 1)^2 + 3 \)
6) \( y = -3(x + 4)^2 + 1 \)

7) \( y = 2(x - 3)^2 + 2 \)
8) \( y = 3(x + 3)^2 - 4 \)

9) \( y = -3(x + 3)^2 - 2 \)
10) \( y = -(x - 2)^2 - 2 \)
Convert each quadratic from Vertex Form to Standard Form. Then solve the quadratic equations.

11) \( y = 2(x - 2)^2 - 2 \)  \hspace{1cm} 12) \( y = -(x - 1)^2 - 3 \)

13) \( y = 2(x + 3)^2 - 2 \)  \hspace{1cm} 14) \( y = -2(x - 2)^2 - 3 \)

15) \( y = (x - 4)^2 + 4 \)  \hspace{1cm} 16) \( y = 3(x - 3)^2 + 1 \)
Vertex Form of Parabolas

Use the information provided to write the vertex form equation of each parabola.

1) \( y = x^2 + 16x + 71 \)

2) \( y = x^2 - 2x - 5 \)

3) \( y = -x^2 - 14x - 59 \)

4) \( y = 2x^2 + 36x + 170 \)

5) \( y = x^2 - 12x + 46 \)

6) \( y = x^2 + 4x \)

7) \( y = x^2 - 6x + 5 \)

8) \( y = (x + 5)(x + 4) \)

9) \( \frac{1}{2}(y + 4) = (x - 7)^2 \)

10) \( 6x^2 + 12x + y + 13 = 0 \)

11) \( 162x + 731 = -y - 9x^2 \)

12) \( x^2 - 12x + y + 40 = 0 \)

13) \( y = x^2 + 10x + 33 \)

14) \( y + 6 = (x + 3)^2 \)
Graphing Quadratic Functions

Complete parts a–c for each quadratic function.

a. Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex.

b. Make a table of values that includes the vertex.

c. Use this information to graph the function.

1. \( f(x) = -2x^2 \)

2. \( f(x) = x^2 - 4x + 4 \)

3. \( f(x) = x^2 - 6x + 8 \)

Determine whether each function has a maximum or a minimum value, and find that value. Then state the domain and range of the function.

4. \( f(x) = 6x^2 \)

5. \( f(x) = -8x^2 \)

6. \( f(x) = x^2 + 2x \)

7. \( f(x) = -2x^2 + 4x - 3 \)

8. \( f(x) = 3x^2 + 12x + 3 \)

9. \( f(x) = 2x^2 + 4x + 1 \)

10. \( f(x) = 3x^2 \)

11. \( f(x) = x^2 + 1 \)

12. \( f(x) = -x^2 + 6x - 15 \)

13. \( f(x) = 2x^2 - 11 \)

14. \( f(x) = x^2 - 10x + 5 \)

15. \( f(x) = -2x^2 + 8x + 7 \)
5-1 Practice

Graphing Quadratic Functions

Complete parts a–c for each quadratic function.

a. Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex.

b. Make a table of values that includes the vertex.

c. Use this information to graph the function.

1. \( f(x) = x^2 - 8x + 15 \)
2. \( f(x) = -x^2 - 4x + 12 \)
3. \( f(x) = 2x^2 - 2x + 1 \)

Determine whether each function has a maximum or minimum value, and find that value. Then state the domain and range of the function.

4. \( f(x) = x^2 + 2x - 8 \)
5. \( f(x) = x^2 - 6x + 14 \)
6. \( v(x) = -x^2 + 14x - 57 \)

7. \( f(x) = 2x^2 + 4x - 6 \)
8. \( f(x) = -x^2 + 4x - 1 \)
9. \( f(x) = -\frac{2}{3}x^2 + 8x - 24 \)

10. GRAVITATION From 4 feet above a swimming pool, Susan throws a ball upward with a velocity of 32 feet per second. The height \( h(t) \) of the ball \( t \) seconds after Susan throws it is given by \( h(t) = -16t^2 + 32t + 4 \). For \( t \geq 0 \), find the maximum height reached by the ball and the time that this height is reached.

11. HEALTH CLUBS Last year, the SportsTime Athletic Club charged $20 to participate in an aerobics class. Seventy people attended the classes. The club wants to increase the class price this year. They expect to lose one customer for each $1 increase in the price.

   a. What price should the club charge to maximize the income from the aerobics classes?

   b. What is the maximum income the SportsTime Athletic Club can expect to make?
5-7 Skills Practice

Transformations with Quadratic Functions

Write each quadratic function in vertex form. Then identify the vertex, axis of symmetry, and direction of opening.

1. \( y = (x - 2)^2 \)  
2. \( y = -x^2 + 4 \)  
3. \( y = x^2 - 6 \)

4. \( y = -3(x + 5)^2 \)  
5. \( y = -5x^2 + 9 \)  
6. \( y = (x - 2)^2 - 18 \)

7. \( y = x^2 - 2x - 5 \)  
8. \( y = x^2 + 6x + 2 \)  
9. \( y = -3x^2 + 24x \)

Graph each function.

10. \( y = (x - 3)^2 - 1 \)  
11. \( y = (x + 1)^2 + 2 \)  
12. \( y = -(x - 4)^2 - 4 \)

13. \( y = -\frac{1}{2}(x + 2)^2 \)  
14. \( y = -3x^2 + 4 \)  
15. \( y = x^2 + 6x + 4 \)
Transformations with Quadratic Functions

Write each equation in vertex form. Then identify the vertex, axis of symmetry, and direction of opening.

1. \( y = -6x^2 - 24x - 25 \)  
2. \( y = 2x^2 + 2 \)  
3. \( y = -4x^2 + 8x \)  
4. \( y = x^2 + 10x + 20 \)  
5. \( y = 2x^2 + 12x + 18 \)  
6. \( y = 3x^2 - 6x + 5 \)  
7. \( y = -2x^2 - 16x - 32 \)  
8. \( y = -3x^2 + 18x - 21 \)  
9. \( y = 2x^2 + 16x + 29 \)

Graph each function.

10. \( y = (x + 3)^2 - 1 \)  
11. \( y = -x^2 + 6x - 5 \)  
12. \( y = 2x^2 - 2x + 1 \)

13. Write an equation for a parabola with vertex at (1, 3) that passes through (–2, –15).

14. Write an equation for a parabola with vertex at (–3, 0) that passes through (3, 18).

15. **BASEBALL** The height \( h \) of a baseball \( t \) seconds after being hit is given by \( h(t) = -16t^2 + 80t + 3 \). What is the maximum height that the baseball reaches, and when does this occur?

16. **SCULPTURE** A modern sculpture in a park contains a parabolic arc that starts at the ground and reaches a maximum height of 10 feet after a horizontal distance of 4 feet. Write a quadratic function in vertex form that describes the shape of the outside of the arc, where \( y \) is the height of a point on the arc and \( x \) is its horizontal distance from the left-hand starting point of the arc.
Part A: Using your calculator as needed, match each equation to its graph.

1. \( y = -(x + 2)(x - 3) \)
2. \( f(x) = (x - 2)(x + 3) \)
3. \( g(x) = (x - 3)(x + 2) \)
4. \( h(x) = (x - 4)(x + 1) \)
5. \( k(x) = (x - 1)(x + 4) \)
6. \( t(x) = -(x - 1)(x + 4) \)
7. \( y = (x - 1)(x - 4) \)
8. \( y = -(x - 1)(x - 4) \)
9. \( f(x) = (x - 0)(x - 4) \)
10. \( g(x) = (x + 2)(x + 2) \)
11. \( h(x) = -(x + 3)(x + 3) \)
12. \( k(x) = (x + 4)(x + 1) \)

What do you see that helps match the equations to their graphs quickly?
Part B: Look carefully at quadratic graphs when the equation is in factored form.

1. Let’s try to make a high quality graph of the $y = (x - 2)(x + 4)$ by hand. Follow these steps.

   a. What’s are the two horizontal intercepts? Plot them on the grid.

   b. Since quadratic graphs are symmetrical, find the line of symmetry by using the horizontal intercepts. Draw it in on the grid.

   c. The vertex lies on the line of symmetry. What is the $x$ value for your line of symmetry? $x = \underline{\hspace{2cm}}$
      Use the equation $y = (x - 2)(x + 4)$ to find the value of $y$ when $x = \underline{\hspace{2cm}}$ (the value of the line of symmetry). Plot this on the grid.

   d. The vertical intercept occurs when $x = 0$. Use the equation $y = (x - 2)(x + 4)$ to find the value of $y$ when $x = 0$. Plot this on the grid.

   e. Using the line of symmetry, find another point that should be part of the graph of $y = (x - 2)(x + 4)$.
      (Hint: the vertical intercept can be reflected.)

   f. Pick another value (like $x = 3$) to find another point.
      Use the equation $y = (x - 2)(x + 4)$ to find the value of $y$ when $x = 3$ (or whatever you chose).
      Plot this on the grid.

   g. Did you plot the symmetric point to the one you just found? If not, do so now.

   h. With your seven points plotted, try to sketch a smooth curve for the graph of $y = (x - 2)(x + 4)$.

2. Use the same strategy as above to try to make a high quality graph of the $f(x) = (x - 5)(x + 1)$ by hand. Be sure to label points on your graph.
3. Make high quality graphs of the following by hand. Label the points you use to create the graph. (You may need a different scale for some.)

\[ g(x) = (x - 1)(x - 7) \]

\[ h(x) = (x + 2)(x - 6) \]

\[ k(x) = -(x + 5)(x - 3) \]

\[ y = -0.5(x + 4)(x - 6) \]
10-2 Skills Practice

Parabolas

Write each equation in standard form. Identify the vertex, axis of symmetry, and direction of opening of the parabola.

1. \( y = x^2 + 2x + 2 \)
2. \( y = x^2 - 2x + 4 \)
3. \( y = x^2 + 4x + 1 \)

4. \( y = -2x^2 + 12x - 14 \)
5. \( x = 3y^2 + 6y - 5 \)
6. \( x + y^2 - 8y = -20 \)

Graph each equation.

4. \( y = (x - 2)^2 \)
5. \( x = (y - 2)^2 + 3 \)
6. \( y = -(x + 3)^2 + 4 \)

Write an equation for each parabola described below. Then graph the equation.

7. vertex (0, 0),
focus \( \left( 0, -\frac{1}{12} \right) \)

8. vertex (5, 1),
focus \( \left( 5, \frac{5}{4} \right) \)

9. vertex (1, 3),
directrix \( x = \frac{7}{8} \)
10-2 Practice

Parabolas

Write each equation in standard form. Identify the vertex, axis of symmetry, and direction of opening of the parabola.

1. \( y = 2x^2 - 12x + 19 \)
2. \( y = \frac{1}{2}x^2 + 3x + \frac{1}{2} \)
3. \( y = -3x^2 - 12x - 7 \)

Graph each equation.

4. \( y = (x - 4)^2 + 3 \)
5. \( x = -\frac{1}{3}y^2 + 1 \)
6. \( x = 3(y + 1)^2 - 3 \)

Write an equation for each parabola described below. Then graph the equation.

7. vertex \((0, -4)\), focus \((0, -3 \frac{7}{8})\)
8. vertex \((-2, 1)\), directrix \(x = -3\)
9. vertex \((1, 3)\), latus rectum: 2 units, \(a < 0\)

10. **TELEVISION** Write the equation in the form \( y = ax^2 \) for a satellite dish. Assume that the bottom of the upward-facing dish passes through \((0, 0)\) and that the distance from the bottom to the focus point is 8 inches.
Writing Equations of Parabolas

Use the information provided to write the vertex form equation of each parabola.

1) Vertex at origin, Focus: \((0, -\frac{1}{32})\)

2) Vertex at origin, Focus: \((0, \frac{1}{8})\)

3) Vertex at origin, Directrix: \(y = \frac{1}{4}\)

4) Vertex at origin, Directrix: \(y = -\frac{1}{8}\)

5) Vertex: \((-5, 8)\), Focus: \((-\frac{21}{4}, 8)\)

6) Vertex: \((-8, -9)\), Focus: \((-\frac{31}{4}, -9)\)

7) Vertex: \((-6, -9)\), Directrix: \(x = -\frac{47}{8}\)

8) Vertex: \((8, 9)\), Directrix: \(y = \frac{73}{8}\)

9) Vertex: \((8, -1)\), y-intercept: \(-17\)

10) Vertex: \((5, -1)\), y-intercept: \(-\frac{27}{2}\)

11) Opens left or right, Vertex: \((7, 6)\), Passes through: \((-11, 9)\)

12) Opens left or right, Vertex: \((7, 0)\), Passes through: \((6, -1)\)

13) Focus: \((-\frac{63}{8}, -7)\), Directrix: \(x = -\frac{65}{8}\)

14) Focus: \((\frac{107}{12}, -7)\), Directrix: \(x = \frac{109}{12}\)
15) Opens up or down, and passes through \((-6, -7), (-11, -2), \text{ and } (-8, 1)\)

16) Opens up or down, and passes through \((11, 15), (7, 7), \text{ and } (4, 22)\)

17)

18)

19) Vertex at origin, opens left, \(\frac{1}{8}\) units between the vertex and focus

20) Vertex at origin, opens right, \(\frac{1}{8}\) units between the vertex and focus

21) Vertex: \((10, 0)\), axis of symmetry: \(y = 0\), length of latus rectum = 1, \(a < 0\)

22) Vertex: \((4, 2)\), axis of symmetry: \(x = 4\), length of latus rectum = \(\frac{1}{3}\), \(a > 0\)

Use the information provided to write the intercept form equation of each parabola.

23) \(x^2 + 3x + y - 28 = 0\)

24) \(-y^2 + x - 20y - 103 = 0\)
Amery recorded the distance and height of a basketball when shooting a free throw.

1. Find the quadratic equation for the relationship of the horizontal distance and the height of the ball. Round to 3 decimal places.

<table>
<thead>
<tr>
<th>Distance (feet), x</th>
<th>Height (feet), f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>8.4</td>
</tr>
<tr>
<td>6</td>
<td>12.1</td>
</tr>
<tr>
<td>9</td>
<td>14.2</td>
</tr>
<tr>
<td>12</td>
<td>13.2</td>
</tr>
<tr>
<td>13</td>
<td>10.5</td>
</tr>
<tr>
<td>15</td>
<td>9.8</td>
</tr>
</tbody>
</table>

2. Using this function what is the approximate maximum height of the ball?

This table shows the population of a city every ten years since 1970.

3. Find the best-fitting quadratic model for the data. Round to 3 decimal places.

<table>
<thead>
<tr>
<th>Years Since 1970, x</th>
<th>Population (In thousands), y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>489</td>
</tr>
<tr>
<td>10</td>
<td>801</td>
</tr>
<tr>
<td>20</td>
<td>1,202</td>
</tr>
<tr>
<td>30</td>
<td>1,998</td>
</tr>
<tr>
<td>40</td>
<td>2,959</td>
</tr>
</tbody>
</table>

4. Using this model, what will be the estimated population in 2020?

5. Which of the following is best modeled by a quadratic function?
   A. Relationship between circumference and diameter.
   B. Relationship between area of a square and side length.
   C. Relationship between diagonal of a square and side length.
   D. Relationship between volume of a cube and side length.

6. If y is a quadratic function of x, which value completes the table?
   A. 12
   B. 20
   C. 44
   D. 48
7. The graph of a quadratic function having the form \( f(x) = ax^2 + bx + c \) passes through the points \((0, -8), (3, 10), \) and \((6, 34)\). What is the value of the function when \( x = -3 \)?

<table>
<thead>
<tr>
<th></th>
<th>A. -32</th>
<th>B. -26</th>
<th>C. -20</th>
<th>D. 10</th>
</tr>
</thead>
</table>

8. Which is the quadratic equation the best fits the scatterplot?

A. \( f(x) = (x - 3)^2 - 4 \)
B. \( f(x) = (x + 3)^2 + 4 \)
C. \( f(x) = (x - 4)^2 - 3 \)
D. \( f(x) = (x + 4)^2 + 3 \)

9. Which is the quadratic equation the best fits the scatterplot?

A. \( f(x) = x^2 - 8x + 22 \)
B. \( f(x) = -x^2 - 8x - 10 \)
C. \( f(x) = -x^2 + 8x - 32 \)
D. \( f(x) = -x^2 + 8x - 10 \)

Write a quadratic equation that fits each set of points.

10. \((0, -8), (2, 0), \) and \((-3, -5)\)

11. \((-1, -16), (2, 5), \) and \((5, 8)\)

12. \((1, 4), (-2, 13), \) and \((0, 3)\)

13. $\begin{array}{|c|c|c|c|c|}
\hline
x & -1 & 0 & 1 & 2 & 3 \\
\hline
y & 35 & 22 & 11 & 2 & -5 \\
\hline
\end{array}$